**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from dr op-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer :We have a normal distribution with = 45 and =8.0. Let X be the amount of time it

take to complete the repair on a customer’s car. To finish in one hour you must have X ≤ 50

so the question is to find pr(X > 50).

Pr(X > 50) = 1 –Pr(X ≤ 50).

Z = (X -)/ = (X -45)/8.0

Thus the question can be answered by using the normal table to find

Pr(X ≤ 50) = Pr (Z ≤(50 – 45)/8.0 = Pr(Z ≤ 0.625) = 73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6%

or 0.2676

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1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Answer : Mean µ = 38

Standard Deviation = 6

Z Score = (Value – Mean)/SD

Z Score for 44 = (44 – 38)/6 = 1 => 84.13%

=> People above 44 age = 100 – 84.13 = 15.87% ≈ 63 out of 400

Z Score for 38 = (38 – 38)/6 = 0 => 50%

Hence People between 38 & 44 age = 84.13 – 50 = 34.13% ≈137 out of 400

Hence More employees at the processing center are older than 44 than between 38

and 44. Is FALSE.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer : Probabilty of employees less than age of 30 = Pr (X < 30).

Z = (X - )/ =(30 - 38)/6

Thus the question can be answered by using the normal table to find

Pr (X ≤ 30) = pr (Z ≤(30 - 38)/6) = Pr(Z ≤ -1.333) = 9.12%

So the number of employees with probability 0.912 of them being under age or

36 employees).

Therefore the statement B of the question is also TRUE.

1. If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid*normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer : As we know that if X ~ N(µ1, σ1^2), and Y ~ N(µ2, σ2^2) are two

Independent random variables then X + Y ~ N(µ1 + µ2, σ1^2), and

X – Y ~ N(µ1 - µ2, σ1^2+σ2^2)

Similarly if Z = aX + by, where X and Y are as defind above, i.e

Z is linear combination of X and Y, then Z ~ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2).

Therefore in the question

2X1 ~ N(2 u, 4 σ^2)and

X1 + X2 ~ N(µ + µ, σ^2 + σ^2) ~ N(2 u, 2σ^2)

2X1 – (X1 + X2) = N(4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and*b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answer :Since we need to find out the values of a and b, which are symmetric about the

Mean, such that the probability of random variable taking a value between them is 0.99,we

Have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is

0.01 (i.e. 1-0.99).

The Probability towards left form a = -0.005(i.e. 0.01/2).

The Probability towards right form b =+ 0.005(i.e. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable

at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), w can calculate the X values.

Z = (X -µ) / σ

For probability 0.005 the Z value is -2.57 (form Z Table).

Z \* σ + µ =X

Z(- 0.005)\*20 + 100 = -(-2.57)\*20 + 100 = 151.4

Z(+0.005)\*20 + 100 = (-2.57)\*20 + 100 =48.6

So, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Answer :

a): Range containing 95% probability for profit of company is

(Rs. 99M, Rs. 1026M).

b): Rs. 170.1 Million.

c): First division of the company has larger probability of making a loss.

Given that:

$1 = Rs.45

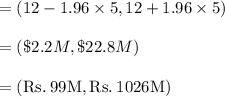
https://tex.z-dn.net/?f=Profit_1%20%5Csim%20N(5%2C%203%5E2)%20%5C%5C%20Profit_2%20%5Csim%20N(7%2C%204%5E2)

Thus,

Company’s profit:

95% of the probability lies between 1.96 standard deviation of the mean.

Thus range is:



1. Specify the 5th percentile of profit (in Rupees) for the company

Answer : Fifth percentile is calculated as :

https://tex.z-dn.net/?f=P(Z%20%5Cleq%20%5Cdfrac%7Bp-12%7D%7B5%7D)%20%3D%200.05

https://tex.z-dn.net/?f=%5Cdfrac%7Bp-12%7D%7B5%7D%20%3D%20-1.644%5C%5Cp%20%3D%2012%20-%208.22%20%3D%203.78%5C%5C

Thus at $3.78M dollars, or Rs. 170.1M amount, 5th  percentile of profit lies.

Or 5th percentile of profit is Rs. 170.1 Million.

1. Which of the two divisions has a larger probability of making a loss in a given year?

Answer :Loss is when profit < 0

Thus: p< 0

The first division of company, thus have larger probability of making a loss in a given

year.